

Dipole-dipole interaction and polarization mode in BEC

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(Dated: January 27, 2013)

We propose the construction of a set of quantum hydrodynamics equations for the Bose-Einstein condensate (BEC) where atoms have electric dipole moment (EDM). The contribution of the dipole-dipole interactions (DDI) to the Euler equation is estimated. Quantum equations for the evolution of medium polarization are constructed for the first time. The mathematical method we developed allows studying the effects of interactions on the evolution of polarization. The developed method may be applied to various physical systems in which dynamics is affected by DDI. A problem of elementary excitations in BEC, either affected or not affected by the uniform external electric field, is addressed using our method. We show that the evolution of polarization in BEC leads to the formation of a novel type of elementary excitations. Also, we consider the process of wave generation in polarized BEC by means of monoenergetic beam of neutral polarized particles. We compute the possibilities of the generation of Bogoliubov's modes and polarization modes by the dipole beam.

I. INTRODUCTION

Since obtaining the Bose-Einstein condensate (BEC) in experiment with pairs of alkaline metal atoms, theoretical and experimental investigation of linear waves and nonlinear structures in it have been performed. The interest to the spinor BEC [1–3], the effect of magnetic moment on the BEC evolution [3–6] and the influence of electrical polarization of atoms on the dynamic processes in BEC [3, 7, 8] is rising in recent years.

Many processes in quantum systems are determined by the dynamics and dispersion of elementary excitations (EE) [9]. A law of dispersion of EE in degenerate dilute Bose gas was obtained by Bogoliubov in 1947 [10, 11]. Later, many authors studied the change of the Bogoliubov's mode which arise when interactions are more carefully in [12, 13], a geometry of the system are complex [14, 15]. In article [7] authors studied the influence of electric dipole moment (EDM) dynamics on dispersion of EE in BEC. The contribution of polarization in dispersion law of Bogoliubov's mode was obtained in [7]. The polarization waves in low dimensional and multy-layer systems of conductors and dielectrics are considered in the papers [16, 17]. Consequently, in BEC of atoms with EDM we expect the existence of polarizations wave; along with Bogoliubov's mode. It is particularly important to take into account the dynamics of EDM in investigations of exciton BEC in semiconductors. The method of QHD [18] was used in investigations of BEC [13] and polarization waves in conductors and semiconductors [16]. Therefore, the method of QHD could be used to consider the possibility of arising polarization waves in BEC and calculating of the dispersion law of these waves.

For investigation of BEC, in system of particles with EDM or magnetic moments, various theoretical methods are used. The model equations, proposed for the description of spinor and polarized BEC (such as the non-linear Schrödinger equation (NLSE) [1, 3, 5, 7]) are generalizations of the Gross-Pitaevskii equation [11].

Other methods were applied for investigation of the polarized BEC and other systems of particles, with EDM, along with NLSE. A hydrodynamic formulation of the Hartree-Fock theory for particles with significant EDM is considered in [19]. In paper [19] the Euler-type equation was derived, from the evolution of the density matrix. The EDM dynamics in dimer Mott insulators causes the rise of the low-frequency mode [20]. A two-body quantum problem for polarized molecules is analyzed in [21]. The existence of states with spontaneous interlayer coherence has been predicted in [22] in systems of polar molecules. Calculations, in [22], were based upon the secondary quantization approach, where the Hamiltonian accounts for the molecular rotation and dipole-dipole interactions. Superfluidity anisotropy of polarized fermion systems was shown in [23] and their thermodynamic and correlation properties were investigated [24]. The effect of EDM in a system of cooled neutral atoms which are used for quantum computing and quantum memory devices was analyzed in [25].

The characteristic property of BEC in a system of excitons inducing in semiconductors [26] it is a significant value of EDM of excitons. This leads to strong interaction of excitons with external electric field and emergence of the collective DDI in exciton systems. QHD method may be applied to such systems along with quantum kinetics based on the nonequilibrium Green functions [27] or density matrix equations [28].

Notable success has been reached in the Bose condensation of dense gases [29, 30]. Consequently, we need the detailed account of short range interaction in investigations of dynamics of EDM. In this work we account the short range interaction (SRI) up to the third order on interaction radius (TOIR) [13]. This approximation leads to nonlocality of SRI [13, 31].

Electrically polarized BEC can interact with the beam of charged and polarized particles by means of charge-dipole and dipole-dipole interaction. Such interaction leads to transfer of energy from beam to medium and, conse-

quently, to generation of waves. In plasma physics the effect of generation of waves by electron [32] or magnetized neutron [33] beam are well-known. In presented paper we consider similar effect in polarized BEC.

We start from the equations of quantum hydrodynamics. System of QHD equations consists of continuity equation, momentum balance equation, eq. of polarization evolution and eq. of polarization current evolution. We made first principles derivation of this equation. For this purpose we used manyparticle Schrödinger equation. In this article we analytically calculate the dispersion properties of polarized BEC. We have shown that the dynamics of EDM leads to existence of new branch in dispersion law. Consequently, there are the waves of polarization in polarized BEC, along with Bogoliubov's mode. We obtain appropriate contribution to the dispersion of Bogoliubov's mode related to DDI. Then, we consider the process of wave generation in polarized BEC by means of monoenergetic beam of neutral polarized particles. We predict new method of generation of Bogoliubov's mode and a new type of modes, rest of the polarization waves.

This paper is organized as follows. We introduce the model Hamiltonian in Sec. II, and present the momentum balance equation. Farther, in Sec. II, we derive the equation of evolution of polarization and obtain the influence of the interactions on the evolution of polarization. In Sec. III we calculate the dispersion dependence of EE in BEC. The polarization evolution is taken into account. We obtain the contribution of polarization in dispersion of Bogoliubov's mode and show the existence of new wave solution. In Sec. IV we study the wave generation in polarized BEC by means of neutral polarized particle beam. In Sec. V we present the brief summary of our results.

II. THE MODEL

Explicit form of the hamiltonian of considering system in a quasi-static approximation is

$$\hat{H} = \sum_i \left(\frac{1}{2m_i} \hat{\mathbf{p}}_i^2 - d_i^\alpha E_{i,ext}^\alpha \right) + \frac{1}{2} \sum_{i,j \neq i} \left(U_{ij} - d_i^\alpha d_j^\beta G_{ij}^{\alpha\beta} \right). \quad (1)$$

The first term here is the operator for kinetic energy. The second term represents the interaction between the dipole moment d_i^α and the external electrical field. The subsequent terms represent short-range U_{ij} and dipole-dipole interactions between particles, respectively. The Green's function for dipole-dipole interaction is taken as $G_{ij}^{\alpha\beta} = \nabla_i^\alpha \nabla_i^\beta (1/r_{ij})$.

For investigation of EE dynamic in polarized BEC we derive the system of QHD equations. This system of equations consists of the continuity equation, the Euler's equation and for the case of polarized particles the equations of polarization evolution and equation of field. The system of equations is derived by methods described in [13].

The first equation of a QHD equations system is the continuity equation

$$\partial_t n(\mathbf{r}, t) + \partial^\alpha (n(\mathbf{r}, t) v^\alpha(\mathbf{r}, t)) = 0. \quad (2)$$

The momentum balance equation for the polarized BEC has the form

$$\begin{aligned} mn(\mathbf{r}, t)(\partial_t + \mathbf{v} \nabla) v^\alpha(\mathbf{r}, t) + \partial_\beta p^{\alpha\beta}(\mathbf{r}, t) \\ - \frac{\hbar^2}{4m} \partial^\alpha \Delta n(\mathbf{r}, t) + \frac{\hbar^2}{4m} \partial^\beta \left(\frac{\partial^\alpha n(\mathbf{r}, t) \cdot \partial^\beta n(\mathbf{r}, t)}{n(\mathbf{r}, t)} \right) \\ = n(\mathbf{r}, t) \Upsilon \partial^\alpha n(\mathbf{r}, t) + \frac{1}{2} \Upsilon_2 \partial^\alpha \Delta n^2(\mathbf{r}, t) \\ + P^\beta(\mathbf{r}, t) \partial^\alpha E^\beta(\mathbf{r}, t), \end{aligned} \quad (3)$$

where

$$\Upsilon = \frac{4\pi}{3} \int dr(r)^3 \frac{\partial U(r)}{\partial r}, \quad (4)$$

and

$$\Upsilon_2 \equiv \frac{\pi}{30} \int dr(r)^5 \frac{\partial U(r)}{\partial r}. \quad (5)$$

In equation (3) we defined a parameter Υ_2 as (5). This definition differs from the one in the work [13]. Terms proportional to \hbar^2 appear as a result of usage of quantum kinematics. The first two members at the right side of the equation (3) are first terms of expansion of the quantum stress tensor. They occur because of taking into account of the SRI potential U_{ij} . The interaction potential U_{ij} determines the macroscopic interaction constants Υ and Υ_2 . The last two members of the equation (3) describe force fields that affect the dipole moment in a unit of volume as the effect of the external electrical field and the field produced by other dipoles, respectively. The last member is written using the self-consistent field approximation [18]. $p^{\alpha\beta}(\mathbf{r}, t)$ is a tensor of the kinetic pressure, which depends on particles' thermal velocities and does not contribute into the BEC dynamics at temperatures near zero.

Also, we have a field equation

$$\nabla \mathbf{E}(\mathbf{r}, t) = -4\pi \nabla \mathbf{P}(\mathbf{r}, t). \quad (6)$$

In the case particles do not bear the dipole moment, the continuity equation and the momentum balance equation form a closed system of equations. When the dipole moment is taken into account in a momentum balance equation, a new physical value emerges, a polarization vector field $P^\alpha(\mathbf{r}, t)$. This causes system of equations to become incomplete.

Next equation we need for investigation of EE dispersion is the equation of polarization evolution

$$\partial_t P^\alpha(\mathbf{r}, t) + \partial^\beta R^{\alpha\beta}(\mathbf{r}, t) = 0. \quad (7)$$

The equation (7) does not contain information about the effect of the interaction on the polarization evolution. The evolution equation of $R^{\alpha\beta}(\mathbf{r}, t)$ can be constructed by analogy with the above derived equations. Method of the equations derivation is described in Appendix. Using a self-consistent field approximation of the dipole-dipole interaction we obtain an equation for the polarization current $R^{\alpha\beta}(\mathbf{r}, t)$ evolution

$$\begin{aligned} & \partial_t R^{\alpha\beta}(\mathbf{r}, t) + \partial^\gamma \left(R^{\alpha\beta}(\mathbf{r}, t) v^\gamma(\mathbf{r}, t) \right. \\ & \left. + R^{\alpha\gamma}(\mathbf{r}, t) v^\beta(\mathbf{r}, t) - P^\alpha(\mathbf{r}, t) v^\beta(\mathbf{r}, t) v^\gamma(\mathbf{r}, t) \right) \\ & + \frac{1}{m} \partial^\gamma r^{\alpha\beta\gamma}(\mathbf{r}, t) - \frac{\hbar^2}{4m^2} \partial_\beta \Delta P^\alpha(\mathbf{r}, t) \\ & + \frac{\hbar^2}{8m^2} \partial^\gamma \left(\frac{\partial_\beta P^\alpha(\mathbf{r}, t) \partial_\gamma n(\mathbf{r}, t)}{n(\mathbf{r}, t)} + \frac{\partial_\gamma P^\alpha(\mathbf{r}, t) \partial_\beta n(\mathbf{r}, t)}{n(\mathbf{r}, t)} \right) \\ & = \frac{1}{m} \Upsilon \partial^\beta (n(\mathbf{r}, t) P^\alpha(\mathbf{r}, t)) \\ & + \frac{1}{m} \frac{P^\alpha(\mathbf{r}, t) P^\gamma(\mathbf{r}, t)}{n(\mathbf{r}, t)} \partial^\beta E^\gamma(\mathbf{r}, t). \end{aligned} \quad (8)$$

Here $r^{\alpha\beta\gamma}(\mathbf{r}, t)$ represents the contribution of thermal movement of polarized particles into the dynamics of $R^{\alpha\beta}(\mathbf{r}, t)$. As we deal with BEC below, the contribution of $r^{\alpha\beta\gamma}(\mathbf{r}, t)$ may be neglected. The last term of the formula (8) includes both external electrical field and a self-consistent field that particle dipoles create. Used in (8) approximations are described in Appendix.

The first term in right side of Eq. (8) describe the short range interaction.

We can see various interactions are included in the equations (3) and (8) additively. At a short distances among particles acts both SRI and dipole-dipole interaction. At large distances remain only dipole-dipole interaction.

From Eq. (8) we can see that the change of polarization arise from both the dipole-dipole interaction and the short range interaction. The SRI among particle leads to displacement of particles. Consequently, as particles has EDM, where are motion of EDM, e.i. changing of the $R^{\alpha\beta}(\mathbf{r}, t)$.

The terms that are proportional to \hbar^2 are of quantum origin as they are analogs of the Bohm quantum potential in the momentum balance equilibrium.

III. ELEMENTARY EXCITATIONS IN THE POLARIZED BEC

We can analyze the linear dynamics of elemental excitations in the polarized BEC using the QHD equations (2), (3), (6), (7) and (8). Let's assume the system is placed in an external electrical field $\mathbf{E}_0 = E_0 \mathbf{e}_z$. The values of concentration n_0 and polarization $\mathbf{P}_0 = \kappa \mathbf{E}_0$ for the system in an equilibrium state are constant and uniform and its velocity field $v^\alpha(\mathbf{r}, t)$ and tensor $R^{\alpha\beta}(\mathbf{r}, t)$ values are zero.

We consider the small perturbation of equilibrium state like

$$n = n_0 + \delta n, \quad v^\alpha = 0 + v^\alpha,$$

$$P^\alpha = P_0^\alpha + \delta P^\alpha, \quad R^{\alpha\beta} = 0 + \delta R^{\alpha\beta}. \quad (9)$$

Substituting these relations into system of equations (2), (3), (7), (8) and (6) and neglecting nonlinear terms, we obtain a system of linear homogeneous equations in partial derivatives with constant coefficients. Passing to the following representation for small perturbations δf

$$\delta f = f(\omega, \mathbf{k}) \exp(-i\omega + i\mathbf{k}\mathbf{r})$$

yields the homogeneous system of algebraic equations. The electric field strength is assumed to have a nonzero value. Expressing all the quantities entering the system of equations in terms of the electric field, we come to the equation

$$\Lambda \cdot E_z = 0,$$

where

$$\begin{aligned} \Lambda = & \omega^2 - \frac{\hbar^2 k^4}{4m^2} + \frac{\Upsilon k^2 n_0}{2m} + 4\pi\sigma \frac{P_0^2 k^2}{mn_0} \\ & - \frac{2\pi\Upsilon k^4 P_0^2}{m^2\omega^2 - \hbar^2 k^4/4 + m\Upsilon k^2 n_0 - m\Upsilon_2 k^4 n_0}. \end{aligned}$$

In this case, the dispersion equation is

$$\Lambda = 0.$$

Solving this equation with respect to ω^2 we obtain a following results.

The dispersion characteristic for EE in BEC can be expressed in the form of

$$\begin{aligned} \omega^2 = & \frac{1}{2m} \left(-\frac{3}{2} \Upsilon n_0 k^2 + \frac{\hbar^2 k^4}{2m} + \Upsilon_2 n_0 k^4 + 4\pi\sigma \frac{P_0^2 k^2}{n_0} \right. \\ & \left. \pm \sqrt{\left(\frac{1}{2} \Upsilon n_0 k^2 - \Upsilon_2 n_0 k^4 + 4\pi\sigma \frac{P_0^2 k^2}{n_0} \right)^2 - 8\pi\Upsilon k^4 P_0^2} \right) \end{aligned} \quad (10)$$

In contrast to the nonpolarized BEC, where only Bogoliubov's mode exists [10, 11, 13], a new wave solution appears in a polarized system due to the polarization dynamics. Bogoliubov's mode corresponds to a negative solution of the equation (10). New wave solution it is a wave

of polarization. In general case presented by formula (10) the frequency of polarization wave in BEC depend on P_0 and Υ . To investigate the BEC polarization effect on the Bogoliubov's mode and the dispersion characteristic of the new solution we analyze extreme cases of the formula (10).

Formula (10) demonstrates that taking account of the BEC polarization dynamics leads to a new solution.

Let's start with the case when the effect of polarization is low compared to the contribution of short-range effects, i.e. of members proportional to Υ and Υ_2 . If so, then formula (10) takes the form

$$\omega_B^2 = \frac{\hbar^2 k^4}{4m^2} - \frac{\Upsilon n_0 k^2}{m} + \frac{\Upsilon_2 n_0 k^4}{m} - \frac{4\pi P_0^2 k^2}{mn_0} \frac{\Upsilon(\sigma - 1) - 2\sigma \Upsilon_2 k^2}{\Upsilon - 2\Upsilon_2 k^2}, \quad (11)$$

$$\omega_P^2 = \frac{\hbar^2 k^4}{4m^2} - \frac{1}{2} \frac{\Upsilon n_0 k^2}{m} + \frac{4\pi P_0^2 k^2}{mn_0} \frac{\Upsilon(\sigma - 1) - 2\sigma \Upsilon_2 k^2}{\Upsilon - 2\Upsilon_2 k^2}. \quad (12)$$

Here we use indexes "B" for Bogoliubov's mode and "P" for new polarization mode. When deriving formulae (11) and (12) we expanded a sub-radical expression in (10) and took only first two terms of the expansion to estimate the influence of polarization on the wave dispersion.

As it follows from equations (7) and (8), changes in polarization can occur due dipole-dipole interactions as well as to SRI and to quantum Bohm potential, i.e. to members proportional to \hbar^2 . That's the reason for existence of other ways of polarization change when the contribution of equilibrium polarization P_0 is neglectable. In the latter case, taken in a linear approximation, changes in the polarization do not affect the concentration evolution.

Formulae (10) are valid even in the absence of the external electrical field when equilibrium polarization equals zero $P_0 = 0$. Equations (10) in that case take the form

$$\omega_B^2 = \frac{1}{m} \left(\frac{\hbar^2 k^4}{4m} - \Upsilon n_0 k^2 + \Upsilon_2 n_0 k^4 \right), \quad (13)$$

$$\omega_P^2 = \frac{1}{2m} \left(\frac{\hbar^2 k^4}{2m} - \Upsilon n_0 k^2 \right). \quad (14)$$

The waves of polarization could be existed at the absence of external electric field $E_0 = 0$. In this case the equilibrium state is no polarized and dipole direction of particles is distributed accidentally.

If the contribution of equilibrium polarization into BEC dispersion (10) is comparable to the contribution of SRI in the third order of the interaction radius Υ_2 relationships (10) are transformed into

$$\omega_B^2 = \frac{\hbar^2 k^4}{4m^2} - \frac{\Upsilon n_0 k^2}{m} + \frac{\Upsilon_2 n_0 k^4}{m} + 4\pi \frac{P_0^2 k^2}{mn_0}, \quad (15)$$

$$\omega_P^2 = \frac{\hbar^2 k^4}{4m^2} - \frac{1}{2} \frac{\Upsilon n_0 k^2}{m} + 4\pi \frac{P_0^2 k^2}{mn_0} (\sigma - 1). \quad (16)$$

Using Feshbach's resonance [34] we can transform the SRI potential in such a way that $\Upsilon = 0$ while $\Upsilon_2 \neq 0$. Formulas (10) in this situation turn into

$$\omega_B^2 = \frac{\hbar^2 k^4}{4m^2} + \frac{\Upsilon_2 n_0 k^4}{m}, \quad (17)$$

$$\omega_P^2 = \frac{\hbar^2 k^4}{4m^2} + 4\pi \sigma \frac{P_0^2 k^2}{mn_0}. \quad (18)$$

In this paper we primarily focus on the influence of BEC polarization on its dispersion characteristics. So, let's consider the case when the contribution into the dispersion of the SRI at the first order of the interaction radius, i.e. terms proportional to Υ , is comparable to the contribution of polarization, and their total effect is much greater than the contribution of terms proportional to Υ_2 . Here, (10) turns into

$$\omega_B^2 = \frac{\hbar^2 k^4}{4m^2} - \frac{\Upsilon n_0 k^2}{m} \left(\frac{(\sigma - 1)}{\sigma} \right), \quad (19)$$

$$\omega_P^2 = \frac{\hbar^2 k^4}{4m^2} + 4\pi \sigma \frac{P_0^2 k^2}{mn_0} - \frac{1}{2} \frac{\Upsilon n_0 k^2}{m} \left(\frac{(\sigma + 4)}{2\sigma} \right). \quad (20)$$

The the first order of the interaction radius interaction constant for dilute gases has the form

$$\Upsilon = -\frac{4\pi \hbar^2 a}{m},$$

where a is the scattering length SL [11, 13]. The value Υ_2 may be expressed approximately as

$$\Upsilon_2 = -\frac{\theta a^2 \Upsilon}{8},$$

where θ is a constant positive value about 1, which depends on the interatomic interaction potential. Finally, Υ_2 takes the form

$$\Upsilon_2 = -\frac{\pi \theta \hbar^2 a^3}{2m}. \quad (21)$$

IV. GENERATION OF WAVES IN POLARIZED BEC

In that section we consider the process of wave generation in BEC by means of the beam of neutral polarized particles. The interaction between beam and BEC has dipole-dipole origin.

To get the dispersion solution we use the system of QHD equations for each sort of particles (2), (3), (7), (8) and the equation of field (6). The equilibrium state of system is characterized by following values of the BEC parameters:

$$\begin{aligned} n &= n_0 + \delta n, \quad v^\alpha = 0 + v^\alpha, \\ P^\alpha &= P_0^\alpha + \delta P^\alpha, \quad R^{\alpha\beta} = 0 + \delta R^{\alpha\beta} \end{aligned} \quad (22)$$

and values of the beam parameters:

$$\begin{aligned} n_b &= n_{0b} + \delta n_b, \quad v_b^\alpha = U\delta^{z\alpha} + \delta v_b^\alpha, \\ P_b^\alpha &= P_{0b}^\alpha + \delta P_b^\alpha, \quad R_b^{\alpha\beta} = R_{0b}^{\alpha\beta} + \delta R_b^{\alpha\beta}. \end{aligned} \quad (23)$$

The polarization P_0^α is proportional to external electric field E_0^α . We consider the case then $\mathbf{E}_0 = [E_0 \sin \varphi, 0, E_0 \cos \varphi]$. In this case the tensor $R_{0b}^{\alpha\beta}$ has only two unequal to zero elements: $R_{0b}^{zx} = R_{0b} \sin \varphi$ and $R_{0b}^{zz} = R_{0b} \cos \varphi$. For the process, under consideration the dispersion relation is:

$$\begin{aligned} 1 + 4\pi k^2 \left(\frac{P_0^2}{\omega^2 - \frac{\hbar^2 k^4}{4m^2} + \frac{\Upsilon n_0 k^2}{2m}} \times \right. \\ \times \left(\frac{\Upsilon k^2 / (2m^2)}{\omega^2 - \frac{\hbar^2 k^4}{4m^2} + \frac{\Upsilon n_0 k^2}{m} - \frac{\Upsilon_2 n_0 k^4}{m}} - \frac{\sigma}{m n_0} \right) \\ \left. + \frac{1}{(\omega - k_z U)^2 - \frac{\hbar^2 k^4}{4m_b^2}} \times \right. \\ \times \left. \left(\frac{2(\omega - k_z U) P_{0b} k_z (P_{0b} U - R_{0b})}{m_b n_{0b} \left((\omega - k_z U)^2 - \frac{\hbar^2 k^4}{4m_b^2} \right)} - \frac{\sigma_b P_{0b}^2}{m_b n_{0b}} \right) \right) = 0. \end{aligned} \quad (24)$$

Using relation $P_{0b} U - R_{0b} = 0$, we can simplify the equation (24) and obtain

$$\begin{aligned} 1 + \frac{\omega_D^2}{\omega^2 - \omega_1^2} \left(\frac{\Upsilon n_0 k^2 / (2m)}{\omega^2 - \omega_2^2} - \sigma \right) \\ - \frac{\sigma_b \omega_{Db}^2}{(\omega - k_z U)^2 - \frac{\hbar^2 k^4}{4m_b^2}} = 0 \end{aligned} \quad (25)$$

In this formula the following designations are used

$$\omega_1^2 = \frac{\hbar^2 k^4}{4m^2} - \frac{\Upsilon n_0 k^2}{2m}, \quad (26)$$

$$\omega_2^2 = \frac{\hbar^2 k^4}{4m^2} - \frac{\Upsilon n_0 k^2}{m} + \frac{\Upsilon_2 n_0 k^4}{m} \quad (27)$$

and

$$\omega_{Di}^2 = \frac{4\pi P_{0i}^2 k^2}{m_i n_{0i}}, \quad (28)$$

where i is the index of sorts of particles, the BEC or the beam.

The equation (25) has two beam related solutions, in the absence of BEC medium:

$$\omega = k_z U \pm \sqrt{\frac{\hbar^2 k^4}{4m_b^2} + \sigma_b \omega_{Db}^2}. \quad (29)$$

We will consider the possibilities of instabilities for the case of low-density beam, the limit case $\omega_{Db} \sim n_{0b} \rightarrow 0$. In this case we can neglect the last term in square root in (29). The resonance interaction beam with the BEC realizes if

$$k_z U \pm \frac{\hbar k^2}{2m_b} = \omega(k), \quad (30)$$

and could lead to instabilities. The quantity $\omega(k)$ is the dispersion of BEC modes (10). The frequency in this case can be presented in the form

$$\omega = k_z U \pm \frac{\hbar k^2}{2m_b} + \delta \omega. \quad (31)$$

Lets consider two limit cases.

small frequency shift limit

In the limit case

$$\delta \omega \ll \hbar k^2 / m \quad (32)$$

the frequency shift obtain in the form:

$$\delta \omega^2 = \pm \frac{2\sigma_b m_b m^2 \omega_{Db}^2 (\omega^2 - \omega_1^2)^2 (\omega^2 - \omega_2^2)^2}{\omega \omega_D^2 \hbar \Upsilon^2 n_0^2 k^6 W}, \quad (33)$$

where

$$W = 2\omega^2 - \omega_1^2 - \omega_2^2 - \frac{2m\sigma}{\Upsilon n_0 k^2} (\omega^2 - \omega_2^2)^2. \quad (34)$$

and the frequency ω determined with formula (10). The instabilities take place in the case $\delta \omega^2 < 0$. The sign of $\delta \omega^2$ is depend on the sign of W .

For the case resonance interaction of beam with the waves in BEC there are instabilities, for the first beam mode in (29) at $W < 0$ and for the second beam mode in formula (29) if $W > 0$. For the polarization mode W is positive. It means that the interaction of polarization mode with the second beam related mode results in the instability. For the Bogoliubov's mode the sign of W depend on σ .

We can consider the following cases:

- (i) the contribution of equilibrium polarization to $\omega(k)$ is dominant; then $W > 0$;
- (ii) the dominant contribution to $\omega(k)$ results from the term in $\omega(k)$ which is proportional to Υ , and, equilibrium polarization and SPI in TOIR give comparable contribution to $\omega(k)$. In this case there is

$$\sigma_0 = 1 + \frac{\Upsilon_2 n_0 k^4 / m}{\Upsilon n_0 k^2 / (2m) + 2\Upsilon_2 n_0 k^4 / m - 4\omega_D^2}$$

. The sign of W varies at $\sigma = \sigma_0$. The dependence of W from σ is presented in the table 1.

large frequency shift limit

In the limit case

$$\delta\omega \gg \hbar k^2 / m \quad (35)$$

we have

$$\delta\omega = \xi \sqrt[3]{\frac{\sigma_b m^2 \omega_{Db}^2 (\omega^2 - \omega_1^2)^2 (\omega^2 - \omega_2^2)^2}{\omega \omega_D^2 \Upsilon^2 n_0^2 k^4 |W|}}, \quad (36)$$

where ξ equal to $\xi_1 = \sqrt[3]{1}$ for $W > 0$ or $\xi_{-1} = \sqrt[3]{-1}$ for $W < 0$. Evident form of quantities ξ_1 and ξ_{-1} is

$$\xi_{-1} = \left[-1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}\right]$$

and

$$\xi_1 = \left[1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}\right].$$

The considerations concerning the sign of W , reflected in the table 1, are valid also for the limit condition (35).

From the formulas (33) and (36) we can see that external neutral particles beam leads to instabilities for both Bogoliubovs and polarization waves.

TABLE I. In this table the sign of the W are presented for the Bogoliubov's mode when SRI is prevailed.

	$\Upsilon > 0$ (attraction)	$\Upsilon < 0$ (repulsion)
$\sigma > \sigma_0$	+	-
$\sigma < \sigma_0$	-	+

V. CONCLUSION

In this work we developed the method for description of dynamics of polarized BEC. This method accounts for the effect of polarization on changes in the concentration and in velocity field, which are determined in general by the continuity equation and Euler's equation. We derived the evolution equations of the polarization and the polarization current. The equations derived contain information about the influence of the interactions on the polarization evolution. We studied the effect of polarization on the BEC dynamics and the influence of SRI on the polarization evolution. An expression of SRI contribution in the equation of polarization current evolution via concentration, polarization and the SRI potential $\Upsilon = \Upsilon(U_{ij})$ was derived. With the assumption that the state of polarized Bose particles in the form of BEC can be described with some single-particle wave function. Changes in polarization due to SRI are shown to be determined at the first order of the interaction radius by the same interaction constant that occurs in Euler's equation and Gross-Pitaevskii equation.

The dispersion of EE in the polarized BEC was analyzed. In the article, we show that polarization evolution in BEC causes a novel type of waves in BEC. The effect of polarization on the dispersion of the Bogoliubov's mode and the dispersion of a new wave mode were studied.

We show the possibility of wave generation in polarized BEC by means of monoenergetic beam of neutral polarized particles.

APPENDIX

At derivation of system of QHD equations we need to write evident form of Hamiltonian of dipole-dipole interaction. In some works the dynamics of the magnetic dipole moment and of the EDM [3, 35] are analyzed in similar ways. Usual expressions of a Hamiltonian for dipole-dipole interaction are equal for electric and magnetic dipoles:

$$H_{dd} = \frac{\delta^{\alpha\beta} - 3r^\alpha r^\beta / r^2}{r^3} d_1^\alpha d_2^\beta.$$

However it has been shown by Breit [36] that a Hamiltonian for spin-spin interaction, and, as a consequence, for the interaction of magnetic moments contains a term that is proportional to Dirac δ -function $\delta(r_1 - r_2) d_1^\alpha d_2^\beta$. The coefficient of the δ -function has been refined later [37] so that the Hamiltonian is in accord with Maxwell's free equations, such as $\text{div} \mathbf{B} = 0$. The resultant expression for the spin-spin interaction Hamiltonian is:

$$H_{\mu\mu} = \left(4\pi \delta_{\alpha\beta} \delta(\mathbf{r}_{12}) + \nabla_1^\alpha \nabla_1^\beta (1/r_{12}) \right) \mu_1^\alpha \mu_2^\beta.$$

Thus, the differences in the dipole-dipole interactions of electric dipoles, and magnetic dipoles, must be taken into account in the development of theoretical field apparatus.

The Schrödinger equation defines wave function in a 3N-dimensional configuration space. Physical processes in systems that involve large number of bodies occur in a three-dimensional physical space [38]. That's why a problem evolves of obtaining a quantum-mechanical description of a system of particles in terms of material fields e.g. concentration, momentum density, energy density and other fields of various tensor dimension that are defined in a three-dimension space.

In equations (3), (6) polarization occurs in the form of

$$P^\alpha(\mathbf{r}, t) = \int dR \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \psi^*(R, t) \hat{d}_i^\alpha \psi(R, t), \quad (37)$$

where \mathbf{r}_i is the coordinate operator for i-th particle, $dR = \prod_{p=1}^N d\mathbf{r}_p$.

In right side of equation (8) where is the force-like field $F^{\alpha\beta}(\mathbf{r}, t)$ which give rise to evolution of the polarization current $R^{\alpha\beta}(\mathbf{r}, t)$. In general form, for $F^{\alpha\beta}(\mathbf{r}, t)$, we can write:

$$F^{\alpha\beta}(\mathbf{r}, t) = -\frac{1}{m} \partial_\gamma \Sigma^{\alpha\beta\gamma}(\mathbf{r}, t) + \frac{1}{m} D^{\alpha\gamma}(\mathbf{r}, t) \partial^\beta E^\gamma(\mathbf{r}, t). \quad (38)$$

A tensor

$$D^{\alpha\beta}(\mathbf{r}, t) = \int dR \sum_i \delta(\mathbf{r} - \mathbf{r}_i) d_i^\alpha d_i^\beta \psi^*(R, t) \psi(R, t), \quad (39)$$

occurs in the term that represents a dipole-dipole interaction and an interaction of the dipole with external electrical field. Based on the reasons of dimensions this value can be approximately presented as

$$D^{\alpha\beta}(\mathbf{r}, t) = \sigma \frac{P^\alpha(\mathbf{r}, t) P^\beta(\mathbf{r}, t)}{n(\mathbf{r}, t)}. \quad (40)$$

A SRI causes the tensor $\Sigma^{\alpha\beta\gamma}(\mathbf{r}, t)$ to occur in the equation (8). Taken at the first order of the interaction radius it has the form

$$\begin{aligned} \Sigma^{\alpha\beta\gamma}(\mathbf{r}, t) = & -\frac{1}{2} \int dR \sum_{i,j \neq i} \delta(\mathbf{r} - \mathbf{R}_{ij}) \\ & \times \frac{r_{ij}^\beta r_{ij}^\gamma}{r_{ij}} \frac{\partial U_{ij}}{\partial r_{ij}} \psi^*(R, t) \hat{d}_i^\alpha \psi(R, t). \end{aligned} \quad (41)$$

Tensor $\Sigma^{\alpha\beta\gamma}(\mathbf{r}, t)$ describe the influence of SRI on evolution of polarization. Eq. (41) describe the SRI.

If we apply the procedure described in [13] to the calculation of the quantum stress tensor, and neglect the contribution of thermal excitations, $\Sigma^{\alpha\beta\gamma}(\mathbf{r}, t)$ for BEC takes a form of

$$\Sigma_{BEC}^{\alpha\beta\gamma}(\mathbf{r}, t) = -\frac{1}{2} \Upsilon \delta^{\beta\gamma} n(\mathbf{r}, t) P^\alpha(\mathbf{r}, t). \quad (42)$$

Formula (42) is obtained for the case where particles located in state with the lowest energy, which could be described by one particle wave function. This state may be the product of strong interaction.

Tensor $\Sigma^{\alpha\beta\gamma}(\mathbf{r}, t)$ is, therefore, like the quantum stress tensor $\sigma^{\alpha\beta}(\mathbf{r}, t)$ in the momentum balance equation (3), dependent on Υ at the first order of the interaction radius [13].

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